Relativistic approach of the LISA mission

B. Chauvineau, S. Pireaux*, T. Régimbau, J.-Y. Vinet

UMR ARTEMIS, Observatoire de la Côte d'Azur, France

sophie.pireaux@obs-azur.fr - tel. +33 (0)4 93 40 53 70

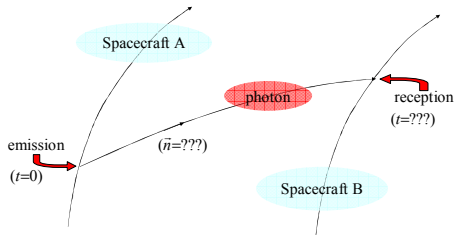
* S. Pireaux acknowledges a CNES postdoctoral grant

[B. Chauvineau, T. Régimbau, J.-Y. Vinet, S. Pireaux, Phys. Rev. D 72, 122003 (2005)]

ABSTRACT

The three LISA spacecraft (numbered 1,2,3) aim at the detection of gravitational waves in the $[10^{-4},10^{-1}]$ Hz band. They are to be launched in 2014, 5 million km appart, in a triangular configuration.

We present a work in progress dealing with the development of a simulator for the LISA mission. This requires a description of orbitography and optical links in a native relativistic framework. The corresponding time delays, crucial for the TDI (Time Delay Interferometry), are provided including Sagnac and relativistic Doppler effects.



Spacecraft and photons are moving in a static spacetime, the metric of which is (γ is a PN parameter)

$$ds^2 = -\left[1 - \frac{2GM}{rc^2} + O\left(\left(\frac{GM}{rc^2}\right)^2\right)\right]c^2dt^2 + \left[1 + \gamma\frac{2GM}{rc^2} + O\left(\left(\frac{GM}{rc^2}\right)^2\right)\right](dx^2 + dy^2 + dz^2)$$

Let us consider a photon going from spacecraft A to spacecraft B. The aim is to compute the energy of this photon measured by an inertial observer, up to order 3/2 in the parameter

$$\varepsilon = \frac{GM}{rc^2} \approx 10^{-8}.$$

At this order, the latter metric ensures a correct description of the solar gravitational field. Since the observer is in free fall, its velocity v ($\approx \sqrt{GM/r}$) is of order 1/2 in ε. Both positions and velocities of A and B are assumed known at initial coordinate time $(t=0)$.

ORBIT AND ENERGY OF A PHOTON

Knowledge of the energy of the photon up to order 3/2 requires knowledge of its orbit up to order 1. It is given by $\overset{(\to 1)}{x}{}^i_{ph}(t,n^j) = x_0^i + n^i ct - (\gamma+1)\frac{GM}{c^2}\chi^i(t,n^j)$ with $\chi^i(t,n^j) = P^i(r - r_0) + n^i \ln\frac{\vec{n}.\vec{x}_0 + ct + r}{\vec{n}.\vec{x}_0 + r_0}$ and $P^i = \frac{x_0^i - (\vec{n}.\vec{x}_0)n^i}{r_0^2 - (\vec{n}.\vec{x}_0)^2}$

The three numbers n^j are integration constants, characterizing the propagation direction. x_0^i is the position of the photon at initial coordinate time.

An observer moving at velocity $\vec{v} \equiv d\vec{x}/dt$ measures the energy $E \propto 1 - \frac{\vec{n}.\vec{v}}{c} + \frac{GM}{rc^2} + \frac{v^2}{2c^2} - \left[(2+\gamma)\frac{GM}{rc^2} + \frac{v^2}{2c^2}\right]\frac{\vec{n}.\vec{v}}{c} + (1+\gamma)\frac{GM}{c^2}.\frac{\vec{P}.\vec{v}}{c}.\frac{\vec{n}.\vec{x}}{r}$

This requires :
- $> n^i$ up to order 1,
- $> \vec{v}$ up to order 3/2 (relativistic motion = classical motion at this order, since the metric is static).

The detection time and the constants n^i are obtained from $x^i_{ph}(t,\vec{n}) = x^i_B(t)$ and the normalisation equation $\vec{n}.\vec{n} = 1$.

The detection equation reads $x^i_{A,0} + n^i ct - (1+\gamma)\frac{GM}{c^2}\chi^i(t,\vec{n}) = x^i_{B,0} + v^i_{B,0}t - \frac{GMx^i_{B,0}}{2r^3_{B,0}}t^2$

and the receiver velocity equation up to order 3/2 reads $v^i_B(t) = v^i_{B,0} - \frac{GMx^i_{B,0}}{r^3_{B,0}}t - \frac{GM}{2}\left[\frac{v^i_{B,0}}{r^3_{B,0}} - 3\frac{(\vec{v}_{B,0}.\vec{x}_{B,0})x^i_{B,0}}{r^5_{B,0}}\right]t^2$

showing that knowledge of the velocity up to order 3/2 requires knowledge of the detection time up to order 1/2. However, TDI requires the detection time up to order 1.

SOLUTION

Travel time $\overset{(0)}{t} = \overset{(0)}{t} + \frac{\overset{(0)}{\vec{n}}.\vec{v}_{B,0}}{c}\overset{(0)}{t} + \frac{1}{2}\left[\frac{v^2_{B,0}}{c^2} + \left(\frac{\overset{(0)}{\vec{n}}.\vec{v}_{B,0}}{c}\right)^2\right]\overset{(0)}{t} + \overset{(0)}{n}{}^j\left[(1+\gamma)\frac{GM}{c^3}\chi^j\left(\overset{(0)}{t},\overset{(0)}{\vec{n}}\right) - \frac{GMx^j_{B,0}}{2r^3_{B,0}c}\overset{(0)}{t}{}^2\right]$

« direction » $n^i = \overset{(0)}{n}{}^i + \frac{v^i_{B,0}}{c} - \overset{(0)}{n}{}^i\frac{\overset{(0)}{\vec{n}}.\vec{v}_{B,0}}{c} - \frac{1}{2c^2}\overset{(0)}{n}{}^i\left[v^2_{B,0} - \left(\overset{(0)}{\vec{n}}.\vec{v}_{B,0}\right)^2\right] - \frac{GM\overset{(0)}{t}}{2r^3_{B,0}c}\left[x^i_{B,0} - \left(\overset{(0)}{\vec{n}}.\vec{x}_{B,0}\right)\overset{(0)}{n}{}^i\right] + (1+\gamma)\frac{GM}{c^3\overset{(0)}{t}}\left[\chi^i - \left(\overset{(0)}{\vec{n}}.\vec{\chi}\right)\overset{(0)}{n}{}^i\right]_{\left(\overset{(0)}{t},\overset{(0)}{\vec{n}}\right)}$

with $c\overset{(0)}{t} = \pm\sqrt{(x^1_{B,0} - x^1_{A,0})(x^1_{B,0} - x^1_{A,0})}$ and $\overset{(0)}{n}{}^j = \frac{x^j_{B,0} - x^j_{A,0}}{c\overset{(0)}{t}}$ (terms of order 0).

The « + » (« - ») sign corresponds to a signal propagating from A to B (B to A).

FREQUENCY SHIFT (photon emitted by A and received by B)

Defined by $1 + z = E_{rec}/E_{em}$. Up to order 3/2 : $\overset{(\to 3/2)}{z} = \overset{(1/2)}{z} + \overset{(1)}{z} + \overset{(3/2)}{z}$

Naively expected order of magnitude $\delta(\varepsilon^k) \approx \varepsilon^k.\frac{L}{2r}$ (L=LISA arm length) $\sim 2.10^{-6}$ $\sim 2.10^{-10}$ $\sim 2.10^{-14}$

In fact, since $\overset{(0)}{\vec{n}}.(\vec{v}_{B,0} - \vec{v}_{A,0}) \approx 0$ (← LISA configuration) $\overset{(1/2)}{z} = -\frac{\overset{(0)}{\vec{n}}.\vec{v}_{B,0} - \vec{v}_{A,0}}{c} \approx 7.10^{-8}$

and $\overset{(1)}{z} = \underbrace{\left(\frac{\overset{(0)}{\vec{n}}.\vec{v}_{B,0} - \vec{v}_{A,0}}{c}\right)^2 - \frac{1}{2}\left(\frac{\vec{v}_{B,0} - \vec{v}_{A,0}}{c}\right)^2}_{\text{Kinematic terms}} + \underbrace{\frac{GM}{c}\frac{\overset{(0)}{t}}{\overset{(0)}{\vec{n}}.\vec{x}_{B,0}}}_{\text{B acceleration during flight of the photon}} + \underbrace{\frac{GM}{c^2}\left(\frac{1}{r_{B,0}} - \frac{1}{r_{A,0}}\right)}_{\text{Einstein gravitational effect}}$

$\sim 2.10^{-15}$ $\sim 6.10^{-12}$ $\sim 2.10^{-10}$ $\sim 2.10^{-10}$

Nearly complete compensation (← LISA configuration)

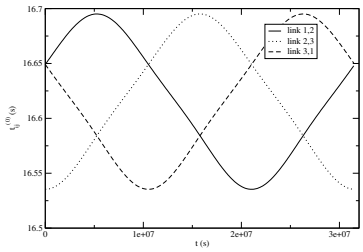
Finally $\overset{(1)}{z} \approx 2.10^{-13}$

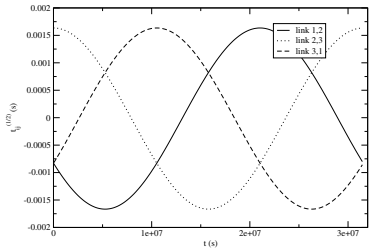
PLANETARY EFFECTS

- During the photon's flight time : centimetric effet → negligible for the laser link
- Should be taken into account for spacecraft ephemerides required to compute flight times (work in progress)

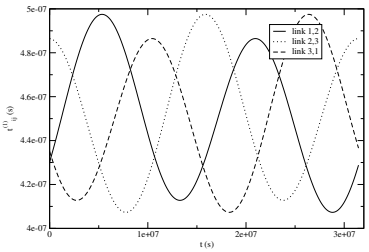
LISACODE, a LISA simulator

Under construction in collaboration with APC/Collège de France (Paris) → simulation of sensitivity curves, TDI tests, ….

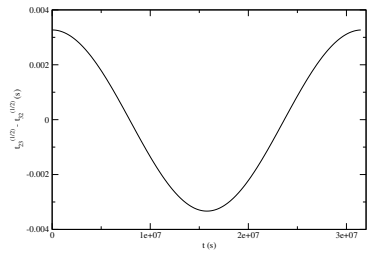
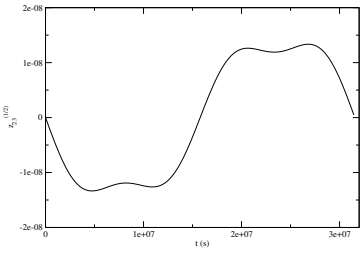
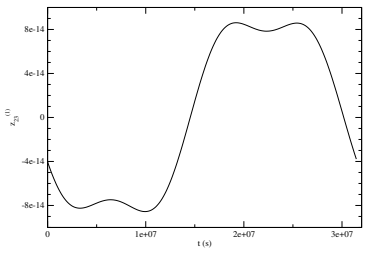




Contributions of orders 0, 1/2 and 1 to the photon travel time from spacecraft A to spacecraft B, over a complete year. The travel time, up to order 1, is the sum of these three contributions.



Sagnac term in the arm 2-3 over a complete year.



Contributions of orders 1/2 and 1 to the frequency shift in the arm 2-3 over a complete year.