Automatic Bayesian inference for LISA data analysis

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Purpose of the talk

Object: - data analysis strategies

Goals: - analysis of unknown number of partially overlapping Gravitational Waves

Method: - Bayes' theorem
- automatic Reversible Jump Markov Chain Monte Carlo (RJMCMC) simulations◆

Outline: Data Analysis, Application, Discussion

Data Analysis

Bayes' Theorem

Prior Likelihood

$$p(K,\vec{\Theta}_K|D) = \frac{p(K)p(\vec{\Theta}_K|K)p(D|K,\vec{\Theta}_K)}{\sum_{K'}\int p(K')p(\vec{\Theta}'_{K'}|K')p(D|K',\vec{\Theta}'_{K'})\mathrm{d}\vec{\Theta}'_{K'}}$$

Posterior Normalisation

D = Data

K = number of signals

$\vec{\Theta}_K$ = parameters of signals + noise

Data Analysis

Goal:

- find joint posterior $p(K,\vec{\Theta}_K|D)$

- marginalise joint posterior $p(\Theta_K{}^j\,|K,D)$

Implementation:

- create a Markov chain with states $x = (K, \vec{\Theta}_K)$
- distribute x according to joint posterior (sampling)

Green, *Biometrika* **82**, 711-732 (1995)

LISA: e.g. Umstaetter et al., *PRD* **72**, 022001 (2006)
MCMC: Cornish & Crowder, *PRD* **72**, 043005 (2005)
MCMC: Cornish & Porter, gr-qc/0605085
LIGO: e.g. Christensen et al., *PRD* **70**, 022001 (2004)

Data Analysis

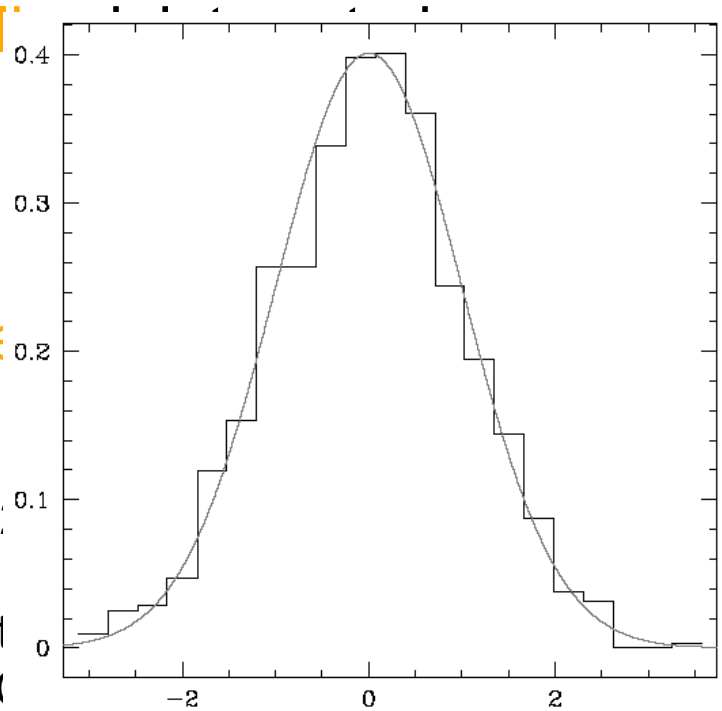
Goal:

- find joint posterior $p(K,\vec{\Theta}_K|D)$

- marginal $(\Theta_K{}^j\,|K,D)$

Implementation:

- create a $x = (K, \vec{\Theta}_K)$
- distribute erior (sampling)



Green, *Biometrika* **8**

LISA: e.g. Umstaett
MCMC: (05 (2005)
MCMC: Cornish & Porter, gr-qc/0605085

LIGO: e.g. Christensen et al., *PRD* **70**, 022001 (2004)

Data Analysis

Technique:

- Metropolis-Hastings sampling
  approximate
  acceptance new states: $\alpha \sim \text{likelihood}(x_{new})/\text{likelihood}(x_{old})$

- Adaption for automation
  Hastie, PhD thesis (2005)
  Atchade/Rosenthal (2003, revised 2005)
  "On Adaptive Markov Chain Monte Carlo Algorithms"

- multiple processor application
  Rosenthal (1999) "Parallel computing and Monte Carlo algorithms"

Application

Data set:
- 2 Michelson Observable , time domain
- LISA orbit and modulations
- Long Wavelength Approximation

Cutler, *PRD* **57**, 7089C (1998)

Signals:
- overlapping sources
- Sources relevant for LISA

→ 1) Binaries: White Dwarf (WD)
→ 2) Binaries: Black Hole + WD
→ 3) EMRI

Goal:
- fit-all-at-once
- find number of signals
- find noise level
- determine joint posterior, marginalised posterior
- determine effect of degree of overlap

Application

Setup:

- 2 signals (no fdot)
- diverse degree of overlap
- search up to 3 signals

- Gaussian white noise

- maximum # parameters: 22
  (7+7+7+1)

- half a year observation time

1) White dwarf binaries

Application

	Setup I		Setup II	
	"ES Cet"	DWD	"ES Cet"	DWD
T_{obs} [sec]	1.55×10^{7}			
Δt [sec]	100			
ρ	80	27	80	27
match	0.02		0.77	
σ_{Noise}	0.1			
A []	1.32×10^{-2}	4.40×10^{-3}	1.30×10^{-2}	2.74×10^{-3}
f [mHz]	3.23	3.23065	3.23	3.23003
Φ_0 [rad]	3.00	3.70	3.00	3.70
θ_N [rad]	0.75	1.93	0.75	0.75
ϕ_N [rad]	5.15	4.30	5.15	5.15
θ_L [rad]	2.21	2.91	2.21	2.21
ϕ_L [rad]	1.25	0.95	1.25	1.25

1) White dwarf binaries

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Application

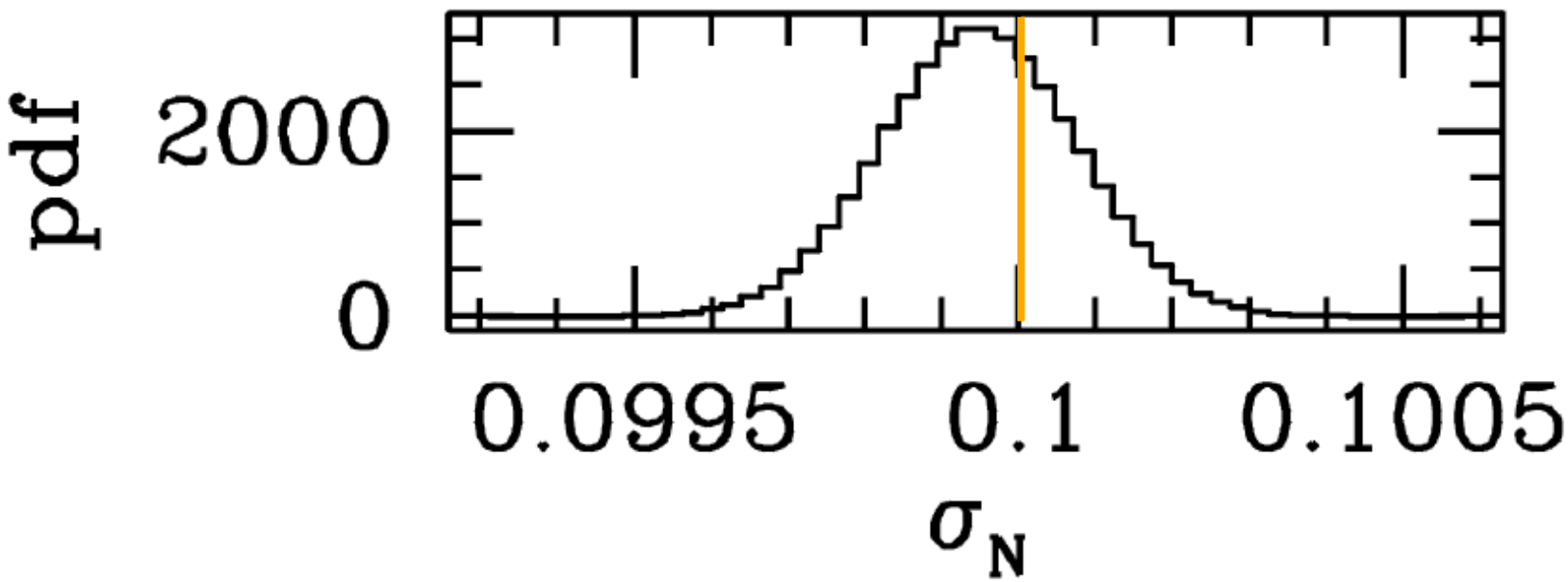
⇒ Result: untangling of signals always successful

Match	0.02	0.77
K=1	0.3 %	0.0 %
K=2	99.7 %	67.3 %
K=3	0.0 %	32.7 %

Application

Result: untangling of signals always successful

⇒ Result: noise level always determined



Application

Result: untangling of signals always successful

Result: noise level always determined

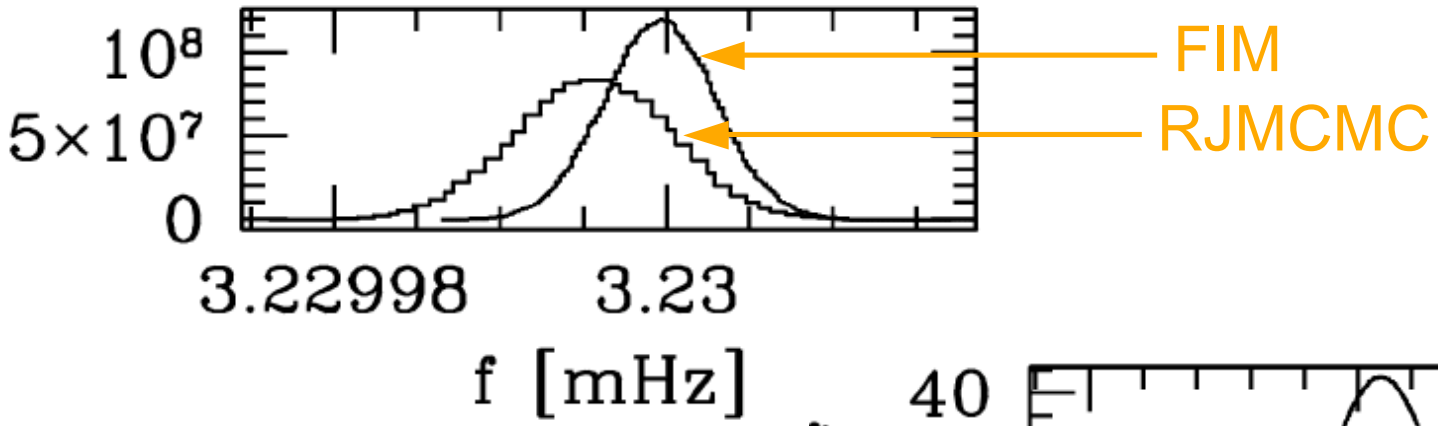
Match 0.02:

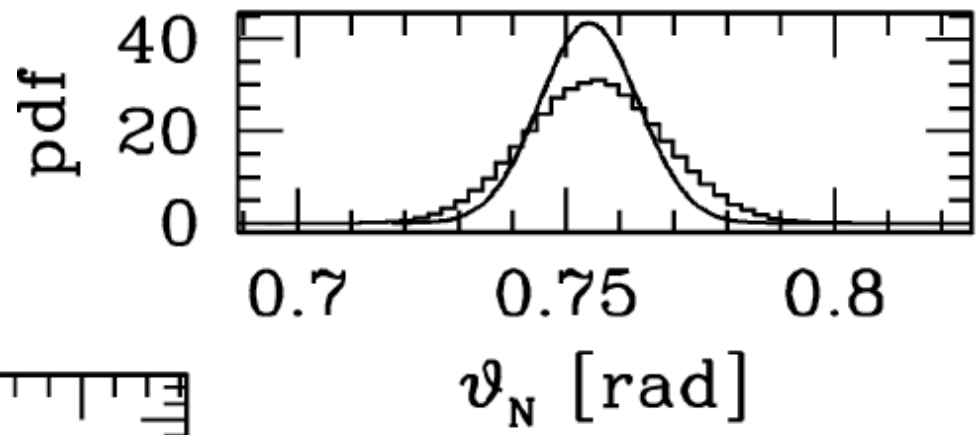
⇒ Result: well matched to predictions by Fisher Information Matrix

⇒ Result: bias / offset

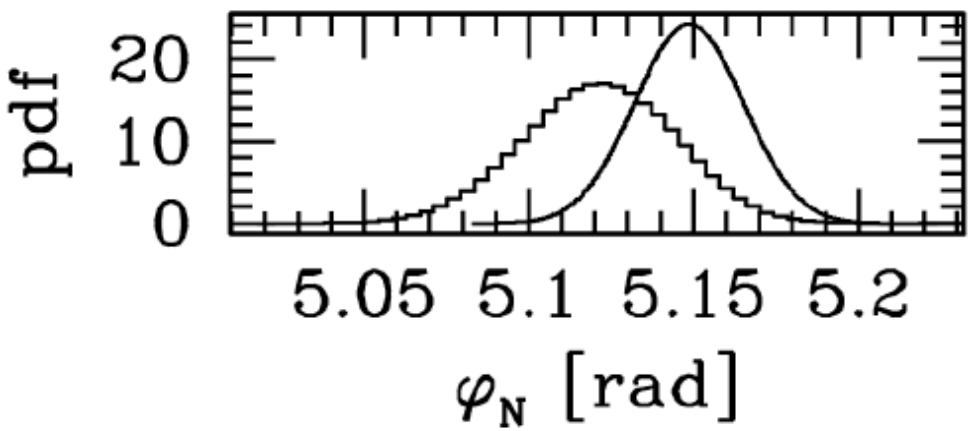
Application

1) White dwarf binaries





"ES Cet": match 0.02



Application

Result: untangling of signals always successful

Result: noise level always determined

Match 0.02:

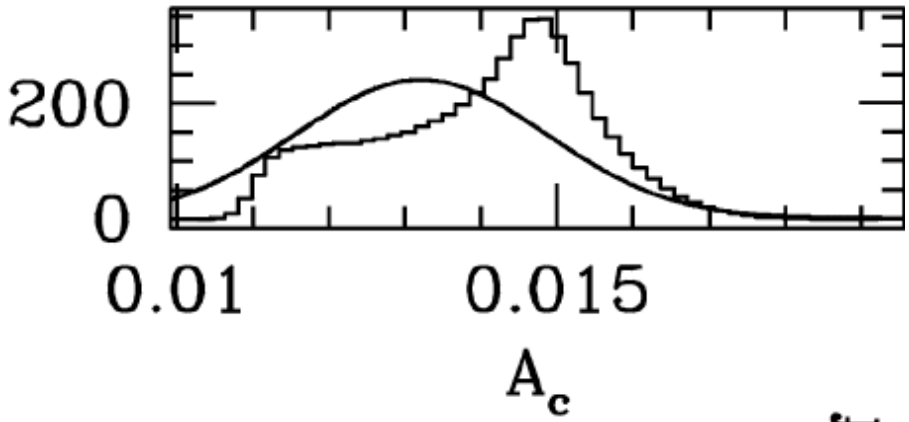
Result: well matched to predictions by Fisher Information Matrix

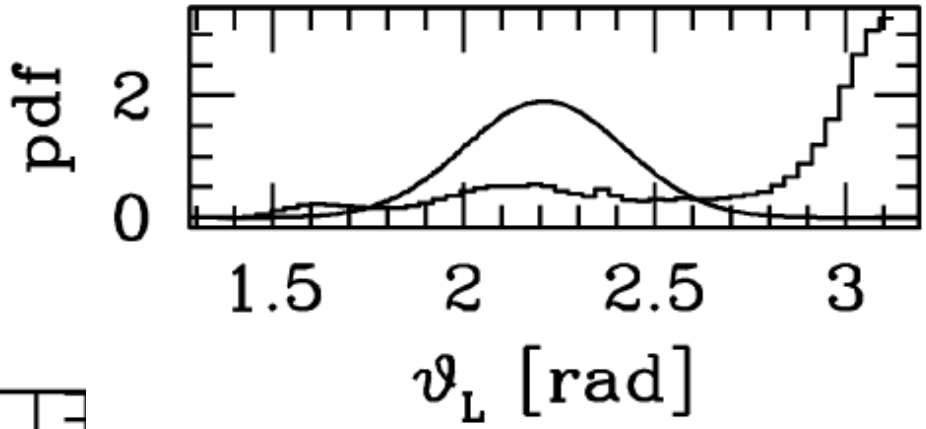
Result: bias / offset

⇒ Result: non-Gaussian posterior in case of correlated parameters

Application

1) White dwarf binaries





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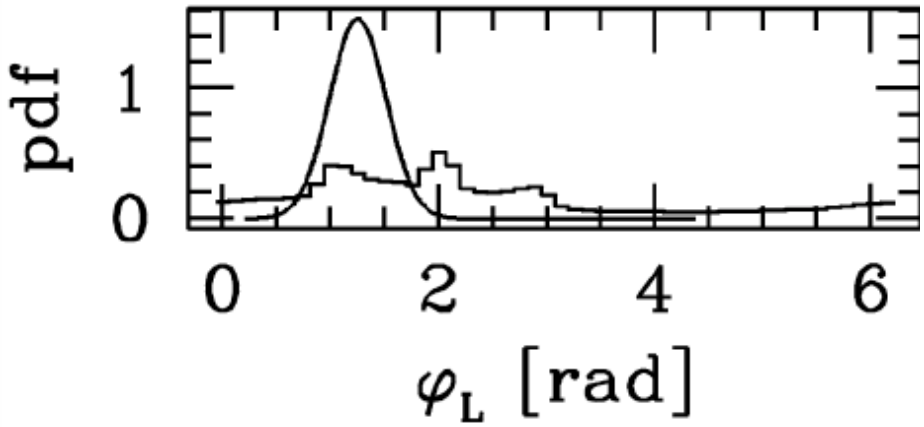


1) White dwarf binaries

Application

Result: untangling of signals always successful

Result: noise level always determined

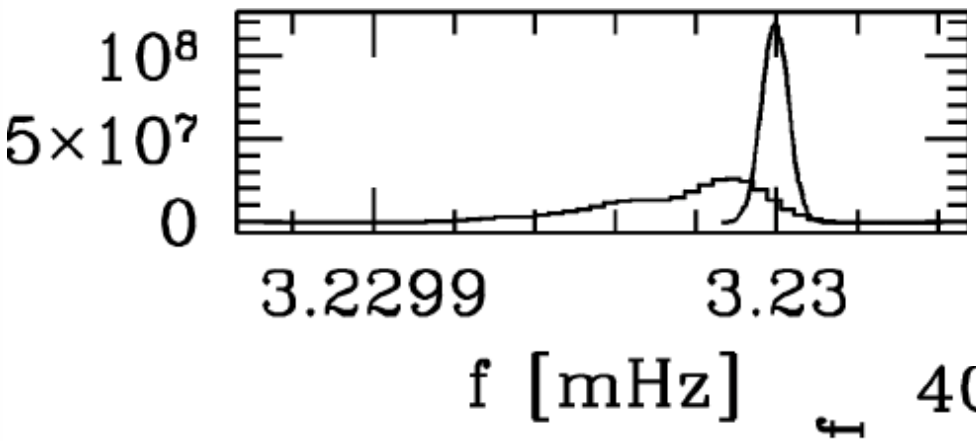
Match 0.77:

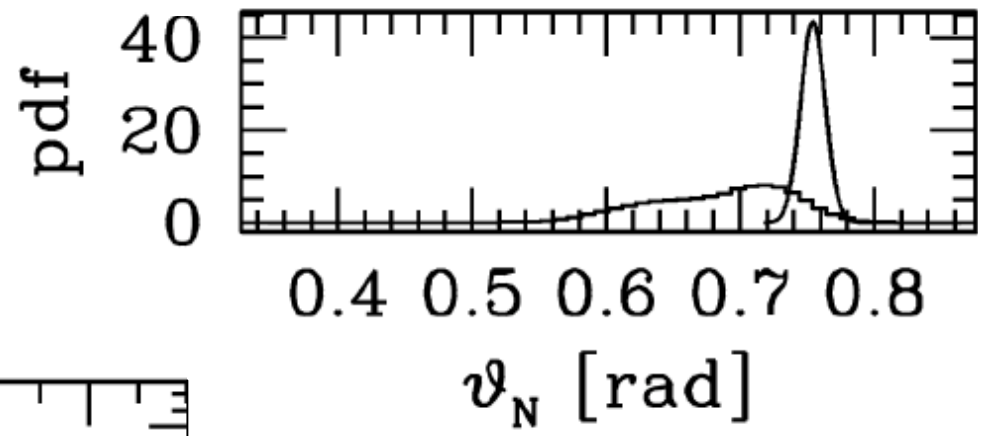
⇒ Result: discrepancy with respect to prediction of Fisher information matrix

⇒ Result: larger bias / offset

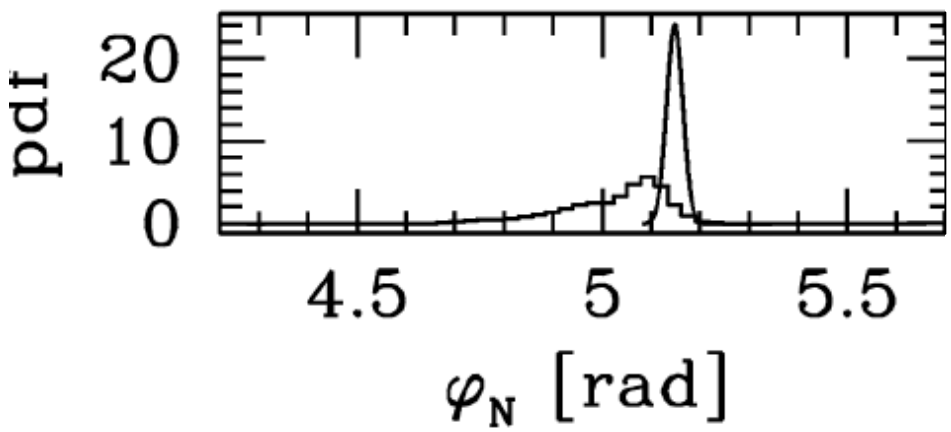
Application

1) White dwarf binaries





"ES Cet": match 0.77



Application

Result: untangling of signals always successful

Result: noise level always determined

Match 0.77:

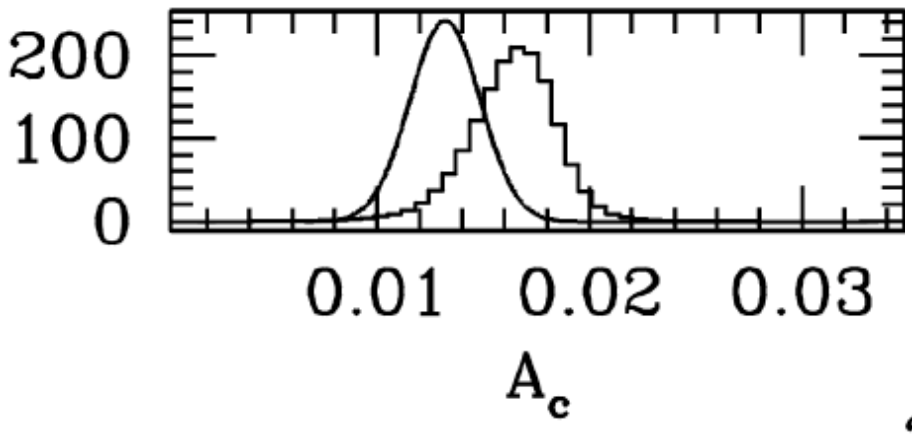
Result: discrepancy with respect to prediction of Fisher information matrix

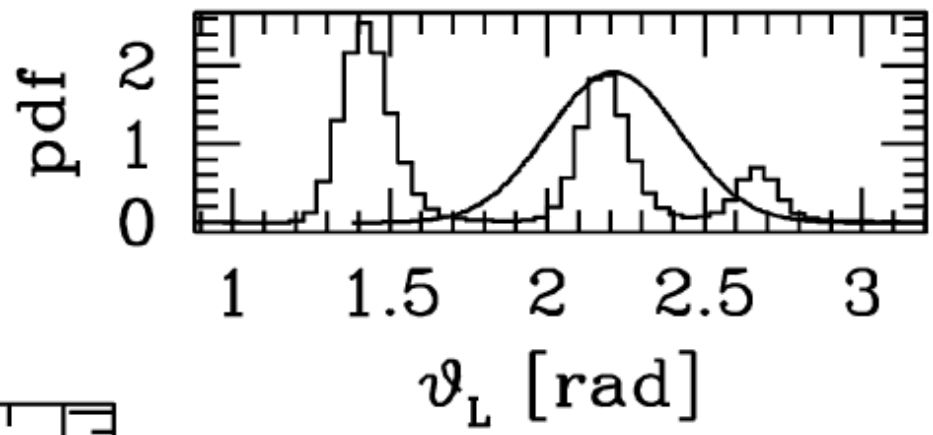
Result: larger bias / offset

⇒ Result: high match and same geometry degrades parameter measurement

Application

1) White dwarf binaries





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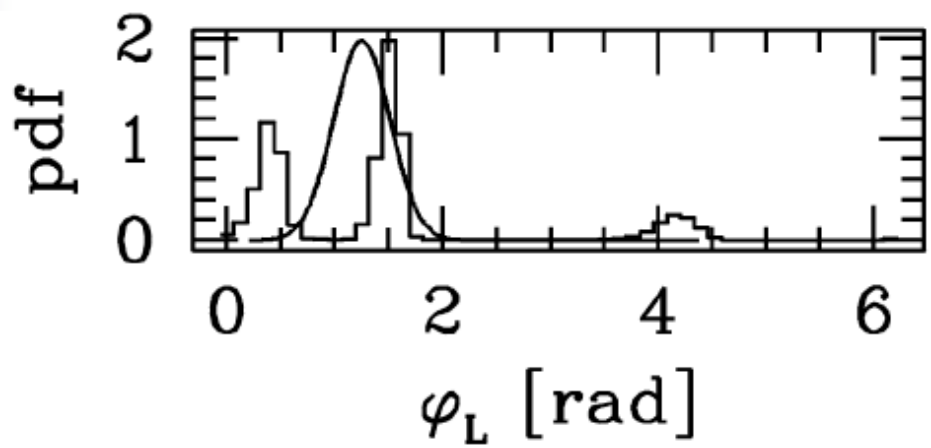


2) Black hole and white dwarf binaries

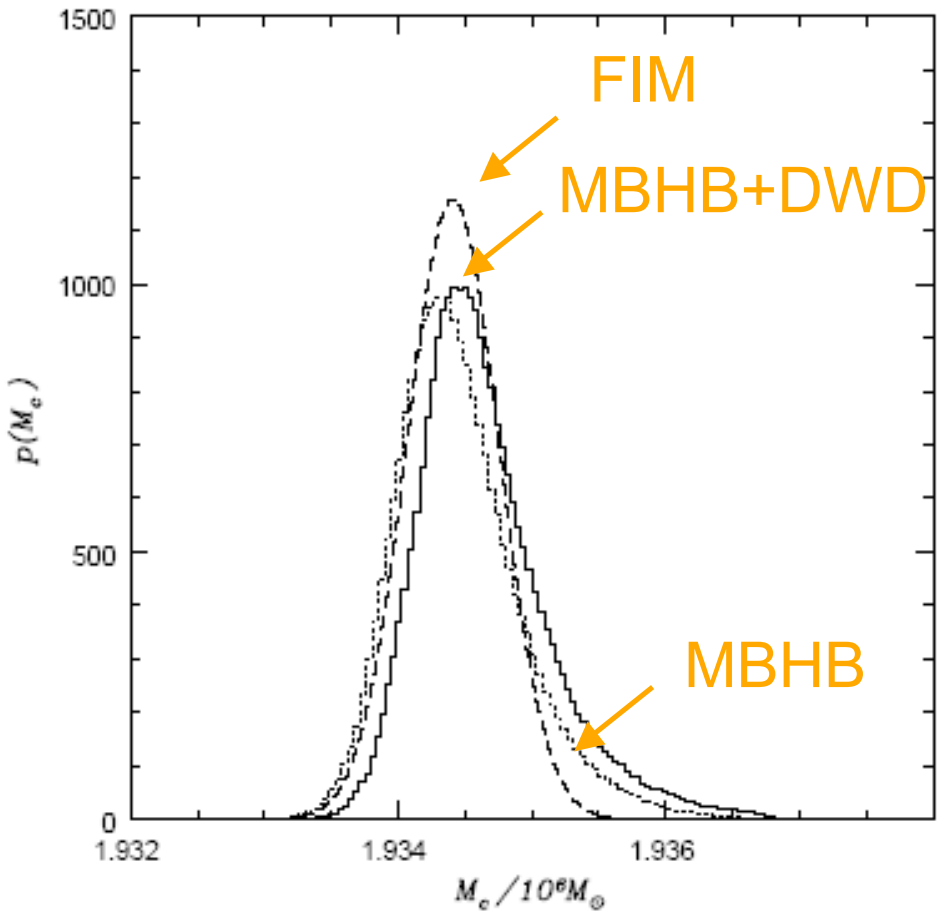
Application

MBHB: $\rho = 250$
(Newtonian)
DWD: $\rho = 45$
(no fdot)

#Parameters: 8+7+1

T_{obs} = 11 days

Result: Untangling of signals successful



Wickham, Stroeer & Vecchio, gr-qc/0605071

3) Extreme mass ratio inspirals

Application

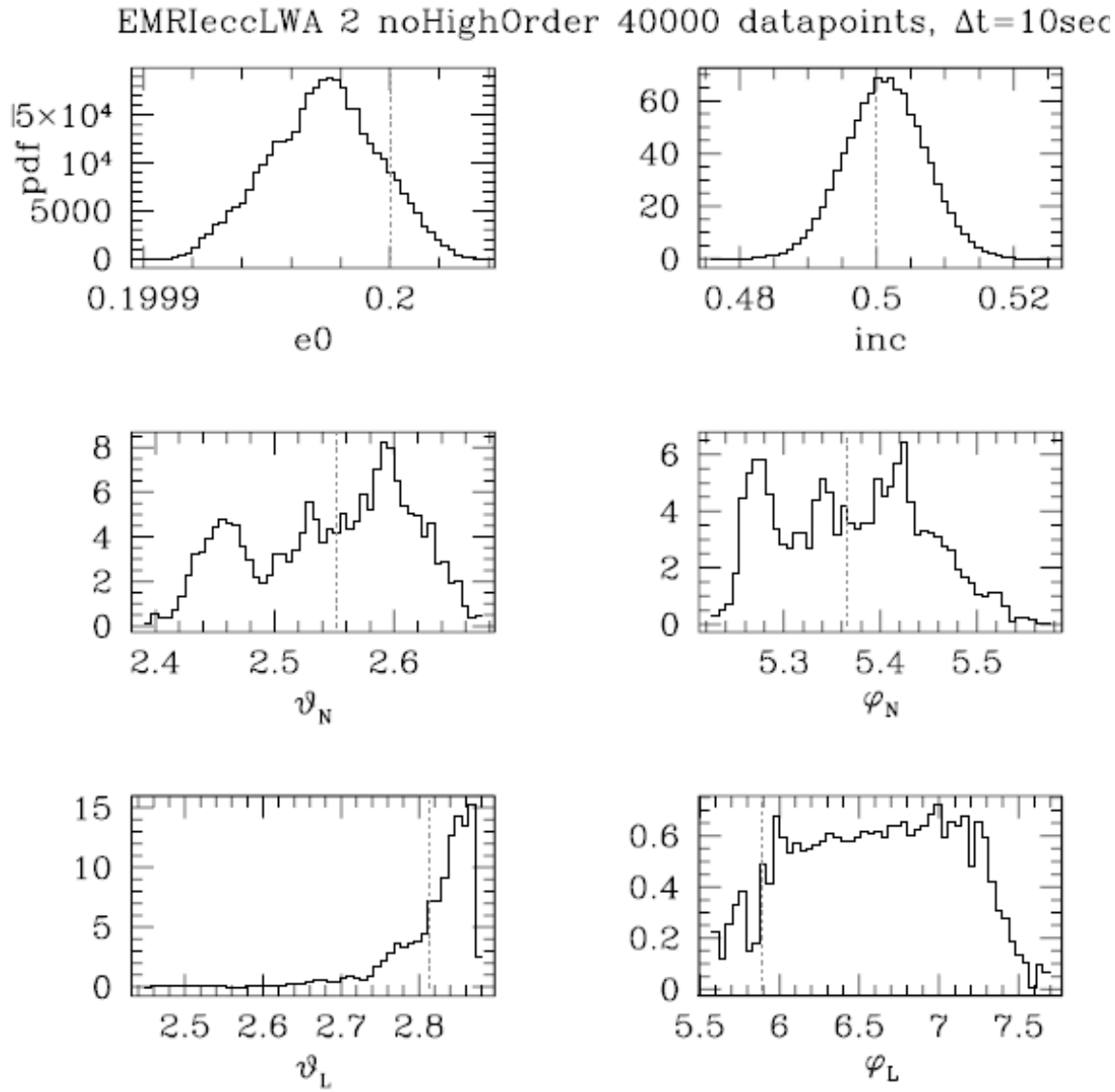
EMRI: ρ = 80
- Simplified Barack & Cutler 04
- e=0.2

T_{obs} = 4 days
tuned $\mu = 10^3 M_\odot$

Parameters: 11+1

Result: Determination of signal successful

In collaboration with J. Gair



Discussion

Advantage:

- automatic approach (user friendly)
- plug and play signal structures / data
- transdimensional approach (# signals not set)

Problems:

- long runtimes
- Bias / Offset

Way forward:

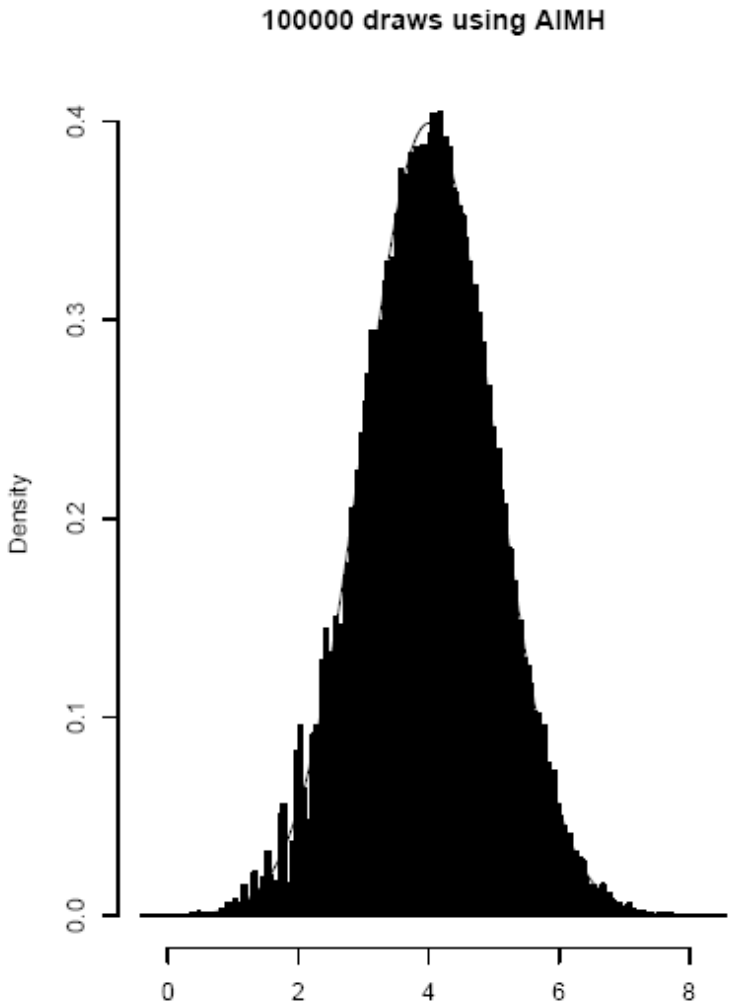
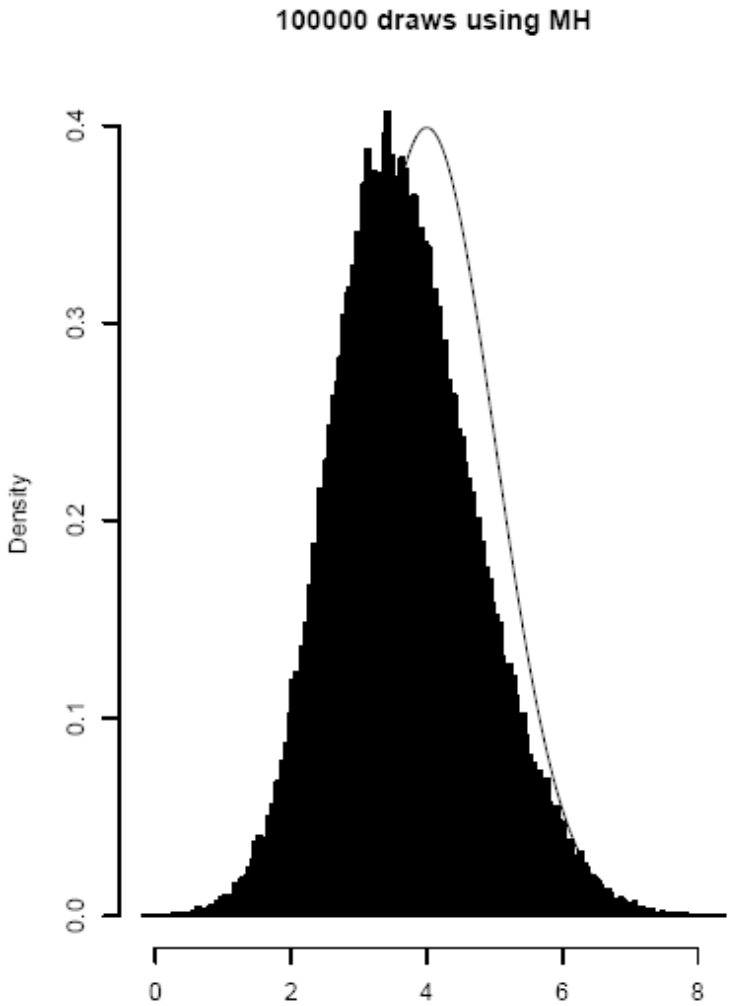
- hybrid Generic Algorithms
- Perfect Sampling

End of talk

Discussion

EMRI simplification:

J. Gair: “We use only the lowest order term in each of the imposed precessions and in the evolution of ν/e, and expand the orbit to fourth order in eccentricity” (email contact)

Discussion

Bias / Offset: